New solitary wave solution of the generalized Hirota-Satsuma couple KdV system

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Abstract: In this research, we find the exact traveling wave solutions involving parameters of the generalized Hirota-Satsuma couple KdV system according to the modified extended tanh-function method with the aid of Maple 16. When these parameters are taken special values, the solitary wave solutions are derived from the exact traveling wave solutions. It is shown that the modified extended tanh-function method provides an effective and a more powerful mathematical tool for solving nonlinear evolution equations in mathematical physics. Comparison between our results and the well-known results will be presented.

Index Terms: The generalized Hirota-Satsuma couple KdV system; The modified extended tanh-function method; traveling wave solutions; solitary wave solutions; dark and bell soliton solutions.

1 INTRODUCTION

No one can deny the important role which played by the nonlinear partial differential equations in the description of many and a wide variety of phenomena not only in physical phenomena, but also in plasma, uid mechanics, optical fibers, solid state physics, chemical kinetics and geochemistry phenomena. So that, during the past ive decades, a lot of method was discovered by a diverse group of scientists to solve the nonlinear partial differential equations. For examples tanh - sech method [15],[20] and [22], extended tanh - method [16], [9] and [24], sine – cosine method [23], [21] and [26], homogeneous balance method [4], the exp($-\phi(\zeta)$)-expansion Method [14], Jacobi elliptic function method [3], [5], [17] and [28], F-expansion method [2], [25] and [12], exp-function method [11] and [10], trigonometric function series method [36], $\left(\frac{G'}{G}\right)$ Expansion method [13], [18], [33] and [30], the modi_ed simple equation method [1], [31], [34], [32], [35] and [29], the modified extended tanh-function method [8], [7], [19], [6] so on.

The objective of this article is to apply the modified extended tanh-function method for finding the exact traveling wave solution of the generalized Hirota-Satsuma couple KdV system [27], which plays an important role in mathematical physics. The rest of this paper is organized as follows: In section 2, we give the description of the modified simple equation method. In section 3, we use this method to find the exact solutions of the nonlinear evolution equations pointed out above. In section 5, conclusions are given.

2 Description of the modified extended tanh-function method

Consider the following nonlinear evolution equation

$$P(u,u_t,u_x,u_{tt},u_{xx},\ldots\ldots)=0, \tag{2.1}$$

Since, P is a polynomial in $u(x,t)$ and its partial derivatives. In the following, we give the main steps of this method

Step 1. We use the traveling wave solution in the form

$$u(x,t)=u(\xi), \qquad \xi=x-ct, \tag{2.2}$$

Where c is a positive constant, to reduce Eq. (2.1) to the following ODE:

$$p(u,u',u'',u''',\ldots\ldots)=0, \tag{2.3}$$

Where P is a polynomial in u (ξ) and its total derivatives, while $u'=\frac{du}{d\xi}$.

Step 2. Suppose that the solution of ODE (2.3) can be expressed

$$u(\xi)=a_0+\sum_{i=0}^{M}\left(a_i\varphi^i+b_i\varphi^{-1}\right), \tag{2.4}$$

where a_i, b_i are arbitrary constants to be determined, such that $a_m \neq 0 \; or \; b_m \neq 0$, while φ satisfies the Riccati equation

$$\varphi'=b+\varphi^2, \tag{2.5}$$

where b is a constant. Eq.(2.5) admits several types of solutions according to

Case 1. If $b<0,$ then

$$\begin{aligned}&\varphi=-\sqrt{-b}\,\tanh\left(\sqrt{-b}\,\xi\right),\\&or\\&\varphi=-\sqrt{-b}\,\coth\left(\sqrt{-b}\,\xi\right).\end{aligned}\tag{2.6}$$

Case 2. If $b>0,$ then

$$\varphi=\sqrt{b}\,\tan\left(\sqrt{b}\,\xi\right),or\ \varphi=\sqrt{b}\,\cot\left(\sqrt{b}\,\xi\right).\tag{2.7}$$

Case 3. If $b=0,$ then

$$\varphi=-\frac{1}{\xi}.\tag{2.8}$$

Step 3. Determine the positive integer m in Eq.(1.4) by balancing the highest order derivatives and the nonlinear terms.

Step 4. Substitute Eq.(1.4) along Eq.(1.5) into Eq.(1.3) and collecting all the terms of the same power $\left(\varphi^i, i=0,\pm1,\pm2,\pm3,\ldots\right)$ and equating them to zero, we obtain a system of algebraic equations, which can be solved by Maple or Mathematica to get the values of a_i and b_i.

Step 5. Substituting these values and the solutions of Eq.(1.5) into Eq.(1.4) we obtain the exact solutions of Eq.(1.1).

3 Application

Here, we will apply the modified extended tanh-function method described in Sec.2 to find the exact traveling wave solutions and the solitary wave solutions of the generalized Hirota-Satsuma couple KdV system[27]. We consider the generalized Hirota-Satsuma couple KdV system

$$\begin{cases}u_t=\frac{1}{4}u_{xxx}+3uu_x+3\left(-v^2+w\right)_x,\\ v_t=-\frac{1}{2}v_{xxx}-3uv_x,\\ w_t=-\frac{1}{2}w_{xxx}-3uw_x.\end{cases}\tag{3.1}$$

When w = 0, Eq. (3.1) reduce to be the well known Hirota-Satsuma couple KdV equation. Using the wave transformation u(x; t) = u (ζ), v(x; t) = v (ζ), w(x; t) = w (ζ), $\zeta=k\left(x-\lambda_1 t\right)$ carries the partial differential equation (3.1) into the ordinary differential equation

$$\begin{cases}-\lambda_1 k\,u'=\frac{1}{4}k^3u'''+3kuu'+3k\left(-v^2+w\right)',\\ -\lambda_1 k\,v'=-\frac{1}{2}k^3v'''-3kuv',\\ -\lambda_1 k\,w'=-\frac{1}{2}k^3w'''-3kuw'.\end{cases}\tag{3.2}$$

Suppose we have the relations between (u and v) and (w and v) $\Rightarrow\left(u=\alpha v^2+\beta v+\gamma\right)$ and (w = Av + B) where α,β,γ,A and B are arbitrary constants. Substituting these relations into second and third equations of Eq. (3.2) and integrating them, we get the same equation and integrate it once again we obtain

$$k^2v'^2=-2\alpha v^4-2\beta v^3+2\left(\lambda_1-3\gamma\right)v^2+2c_1v+c_2,\tag{3.3}$$

Where c_1 and c_2 is the arbitrary constants of integration, and hence, we obtain

$$\begin{aligned}k^2u''&=2\alpha k^2v'^2+k^2\left(2\alpha v+\beta\right)v''\\&=2\alpha\begin{bmatrix}-\alpha v^4-2\beta v^3+2\left(\lambda_1-3\gamma\right)v^2\\+2c_1v+c_2\end{bmatrix}\\&\quad+\left(2\alpha v+\beta\right)\begin{bmatrix}-2\alpha v^3-3\beta v^2\\+2\left(\lambda_1-3\gamma\right)v+c_1\end{bmatrix}.\end{aligned}\tag{3.4}$$

So that, we have

$$P''+\ell P-m\,P^3=0.\tag{3.5}$$

Where

$$c_1=\frac{1}{2\alpha^2\left(\beta^2+2\lambda_1\alpha\beta-6\alpha\beta\gamma\right)},v\left(\zeta\right)=a\,P\left(\zeta\right)-\frac{\beta}{2\alpha},$$

$$\alpha=\frac{\beta^2-4}{4\left(\gamma-\lambda_1\right)},A=\frac{4\beta\left(\lambda_1-\gamma\right)}{\beta^2-4},$$

$$B=\frac{1}{6\left(-\gamma+\lambda_1\right)\left(\beta^2-4\right)^2}\begin{pmatrix}16c_3\lambda_1\beta^2-2c_3\lambda_1\beta^4\\-16c_3\gamma\beta^2+3c_3\gamma\beta^4\\+56\lambda_1^2\gamma\beta^2+-48\gamma^2\lambda_1\beta^2\\-16c_2+c_2\beta^6-12c_2\beta^4\\+12c_2\beta^2-16\gamma^2\lambda_1\\-32\lambda_1^2\gamma-8\lambda_1^3\beta^2+\beta^4\gamma^3\end{pmatrix},$$

$$\ell=\frac{-a}{k^2}\left(\frac{3\beta^2}{2\alpha}+2\lambda_1-6\gamma\right),m=\frac{-2\alpha a^3}{k^2}.$$

Balancing between the highest order derivatives and nonlinear terms appearing in P'' and P^3 $\Rightarrow$

ISSN 2229-5518

(N + 2 = 3N) $\Rightarrow$ (N = 1). So that, by using Eq. (2.4) we get the formal solution of Eq. (3.5)

$$P(\zeta) = a_0 + a_1\phi + \frac{b_1}{\phi}. \tag{3.6}$$

Substituting Eq. (3.6) and its derivative into Eq. (3.5) and collecting all term with the same power of $\phi^3, \phi^2, ..., \phi^{-2}, \phi^{-3}$ we obtained:

$$2a_1 + m\, a_1^3 = 0, \tag{3.7}$$

$$3m\, a_0\, a_1^2 = 0, \tag{3.8}$$

$$2a_1 b + \ell a_1 + 3m\, a_0^2\, a_1 + 3m\, a_1^2\, b_1 = 0, \tag{3.9}$$

$$\ell a_0 + m\, a_0^3 + 6m\, a_0\, a_1\, b_1 = 0, \tag{3.10}$$

$$2b_1 b + \ell b_1 + 3m\, a_0^2\, b_1 + 3m\, a_1\, b_1^2 = 0, \tag{3.11}$$

$$3m\, a_0\, b_1^2 = 0 \tag{3.12}$$

$$2b_1 b^2 + m\, b_1^3 = 0. \tag{3.13}$$

Solving above system of algebraic equations by using Maple 16, we obtain

$$\ell = -8b, a_0 = 0, a_1 = \pm\sqrt{\frac{-2}{m}}, b_1 = \mp\sqrt{\frac{-2}{m}}b.$$

Thus the solution is

$$P(\zeta) = \pm\sqrt{\frac{-2}{m}}\,\phi \mp \sqrt{\frac{-2}{m}}\,\frac{b}{\phi}. \tag{3.14}$$

Sothat let us discuss the following cases:

Case 1. If b < 0, then

$$P(\zeta) = \pm\sqrt{\frac{2b}{m}}\tanh\left(\sqrt{-b}\,\zeta\right) \mp \sqrt{\frac{2b}{m}}\,\frac{1}{\tanh\left(\sqrt{-b}\,\zeta\right)}. \tag{3.15}$$

Or

$$P(\zeta) = \pm\sqrt{\frac{2b}{m}}\coth\left(\sqrt{-b}\,\zeta\right) \mp \sqrt{\frac{2b}{m}}\,\frac{1}{\coth\left(\sqrt{-b}\,\zeta\right)}. \tag{3.16}$$

Case 2. If b > 0, then

$$P(\zeta) = \pm\sqrt{\frac{-2b}{m}}\tan\left(\sqrt{b}\,\zeta\right) \mp \sqrt{\frac{-2b}{m}}\,\frac{1}{\tan\left(\sqrt{b}\,\zeta\right)}. \tag{3.17}$$

Or

$$P(\zeta) = \pm\sqrt{\frac{-2b}{m}}\cot\left(\sqrt{b}\,\zeta\right) \mp \sqrt{\frac{-2b}{m}}\,\frac{1}{\cot\left(\sqrt{b}\,\zeta\right)}. \tag{3.18}$$

Case 3. If b = 0, then

$$P(\zeta) = \pm\sqrt{\frac{-2}{m}}\,\frac{1}{\zeta} \mp \sqrt{\frac{-2}{m}}\,b\,\zeta. \tag{3.19}$$

Note that:

All the obtained results have been checked with Maple 16 by putting them back into the original equation and found correct.

4 Physical Meaning of each solution:

Here, we explain the physical meaning of solution since we can not that:

Eqs.(3.15), (3.16), (3.17), (3.18) and (3.19) depend on some parameters like m; b when this parameters take special values we can draw the solutions and explain what is the mean of this figures.

For example: when $\left(m = -4, b = -2, k = 1, \lambda_1 = 2\right)$

So that Eqs. (3.15) and (3.16) has four figures which represent singular soliton solutions. Also, when $\left(m = -4, b = 2, k = 1, \lambda_1 = 2\right)$ So that Eqs. (3.17) and (3.18) has four figures which represent singular soliton solutions. Also, when $\left(m = -2, b = 2, k = 1, \lambda_1 = 2\right)$ So that Eq. (3.19) has two figures which represent singular soliton solutions.

5 Conclusions

The modified extended tanh-function method has been successfully used to _nd the exact traveling wave solutions of some nonlinear evolution equations. As an application, the traveling wave solutions for the generalized Hirota Satsuma couple KdV system which has been constructed using the modified extended tanh-function method. Let us compare between our results obtained in the present article with the well-known results obtained by other authors using different methods as follows: Our results of the generalized Hirota-Satsuma couple KdV system are new and different from those obtained in [8] It can be concluded that this method is reliable and propose a variety of exact solutions NPDEs. The performance of this method is e_ective and can be applied to many other nonlinear evolution equations. The solutions represent the solitary traveling wave solution for the generalized Hirota-Satsuma couple KdV system.

6 Competing interests

This research received no specific grant from any funding agency in the public, commercial, or not-for-pro_t sectors. The author did not have any competing interests in this research.

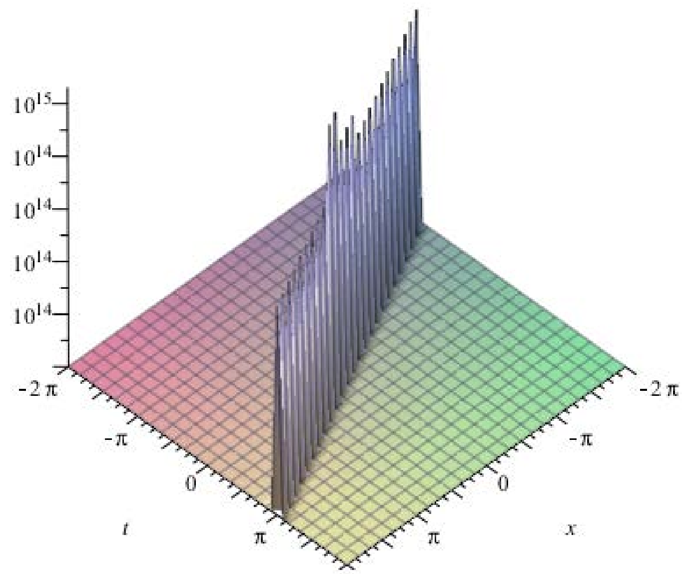
7 Author's contributions

All parts contained in the research carried out by the researcher through hard work and a review of the various references and contributions in the field of mathematics and the physical Applied

8 Acknowledgments

The author thanks the referees for their suggestions and comments.

ISSN 2229-5518



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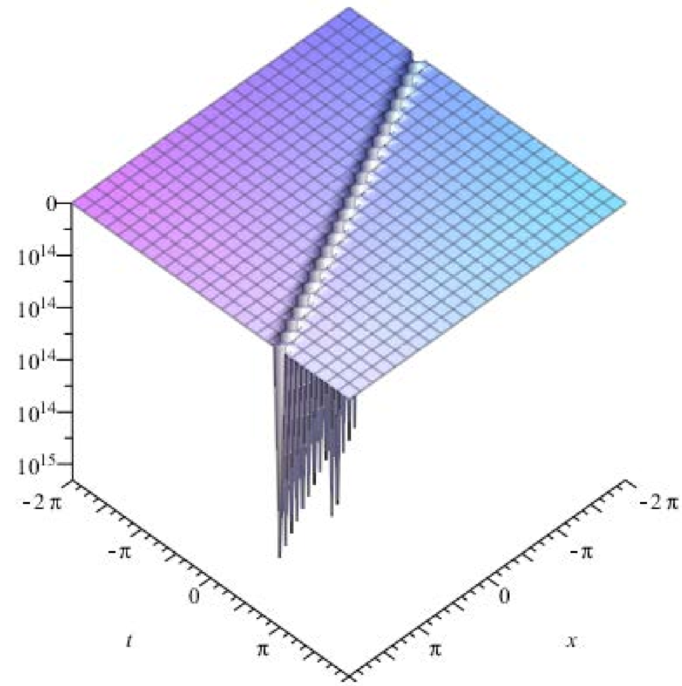

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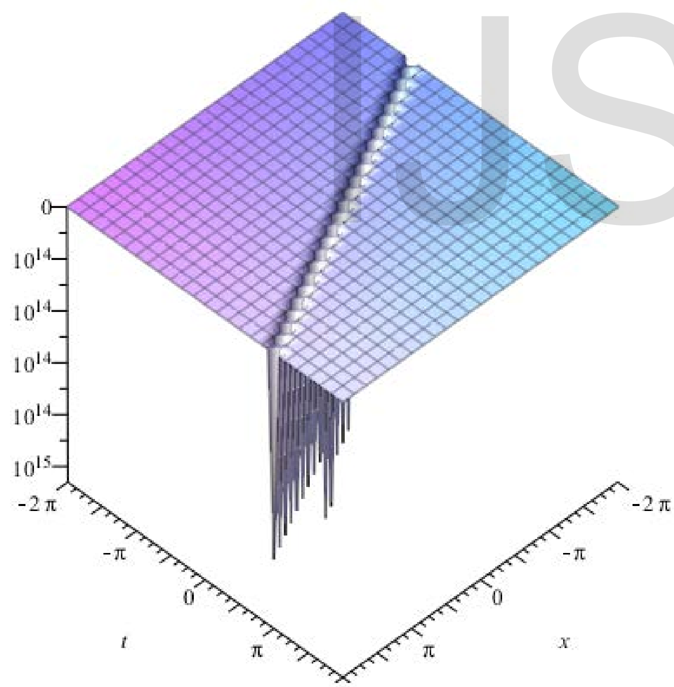

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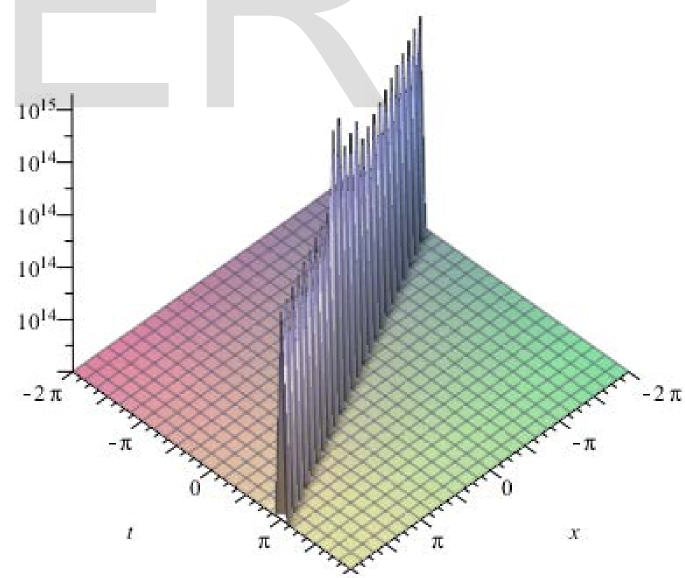

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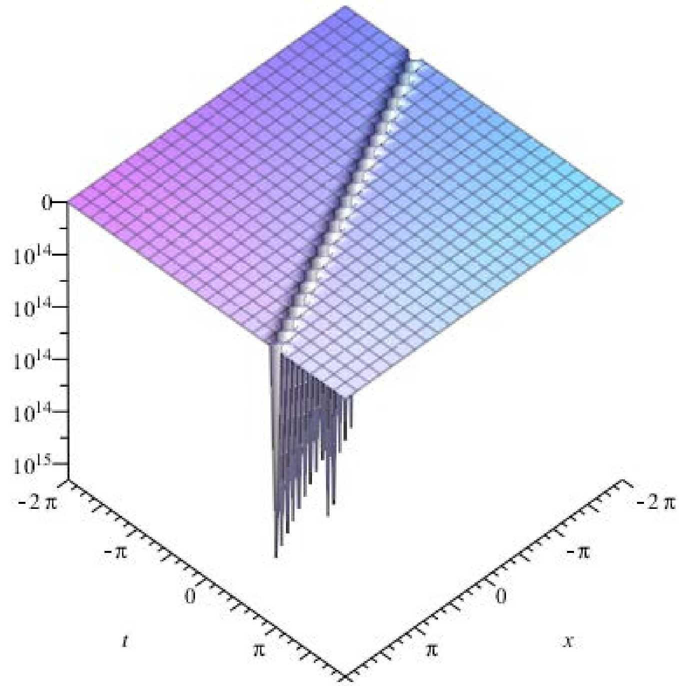
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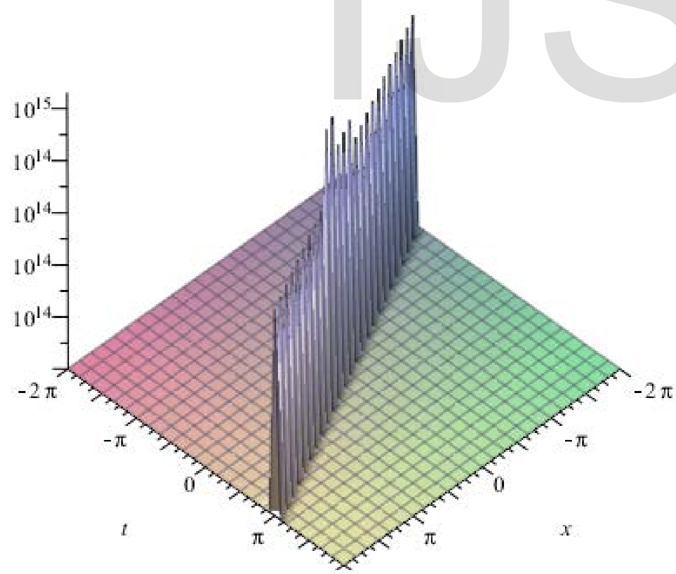
Eq.(3.17)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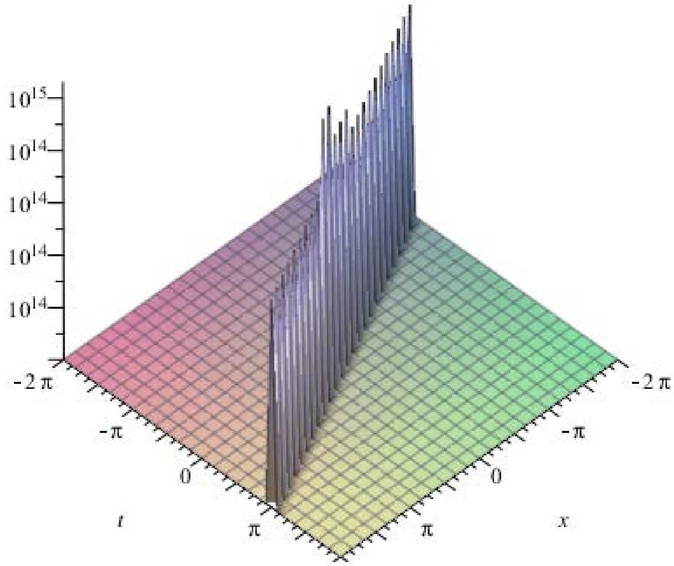
Eq.(3.17)



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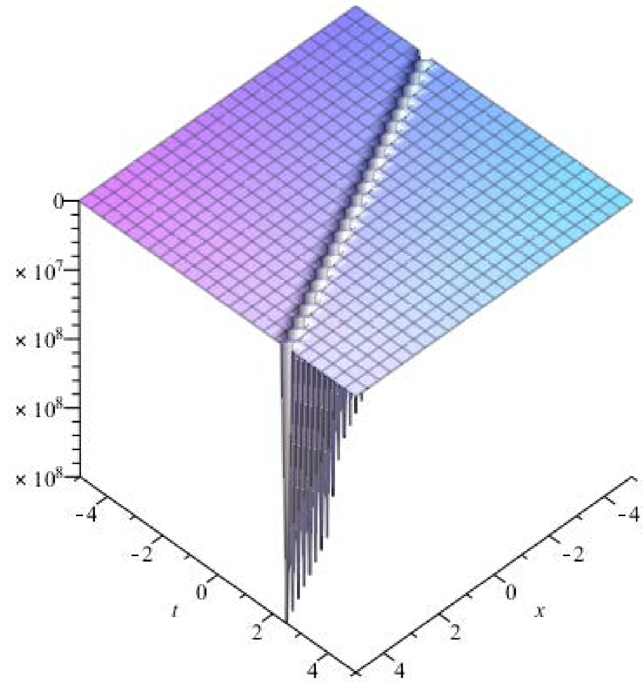

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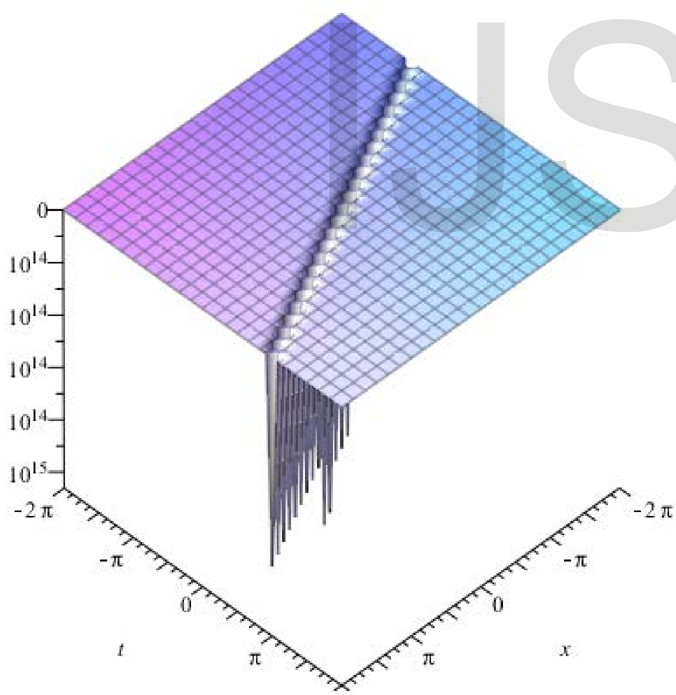

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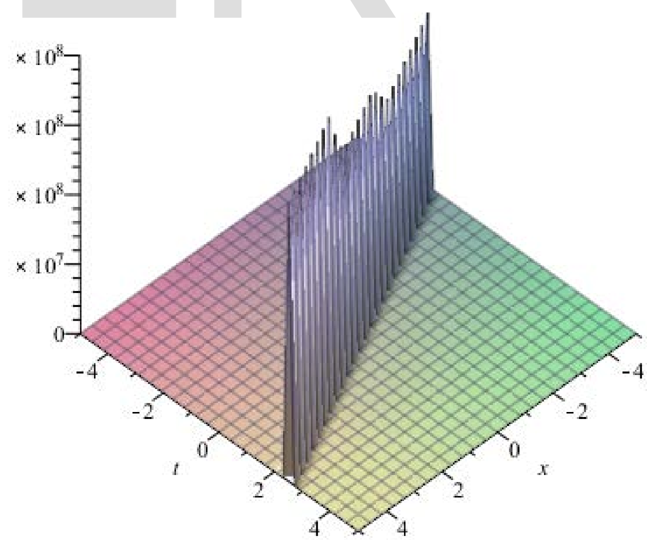

Eq.(3.19)

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